

## Practical work on planar contact

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### Introduction

Let's try to find the stress fields in the vicinity of contact, and displacements on the surface, induced by three specific contact-pressure distributions defined on the interval  $x \in [-a, a]$ :

1. Uniform pressure (Fig. 1(a)):  $p = p_0$
2. Singular at edges pressure (Fig. 1(b)):  $p = \frac{p_0}{\sqrt{1 - x^2/a^2}}$
3. "Elliptic" pressure (Fig. 1(c)):  $p = p_0 \sqrt{1 - x^2/a^2}$

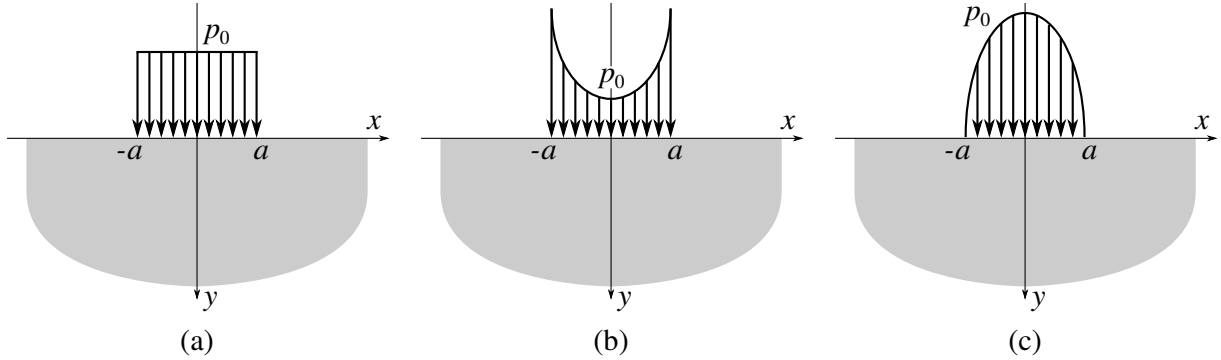


Figure 1: Pressure distribution on the half-space: (a) uniform, (b) singular at edges, (c) "elliptic" or Hertzian pressure.

To find the stress field and the displacement, we will use the following equations (use numerical integration):

$$\sigma_x(x, y) = -\frac{2y}{\pi} \int_{-b}^a \frac{p(s)(x-s)^2 ds}{((x-s)^2 + y^2)^2}$$

$$\sigma_y(x, y) = -\frac{2y^3}{\pi} \int_{-b}^a \frac{p(s) ds}{((x-s)^2 + y^2)^2}$$

$$\sigma_{xy}(x, y) = -\frac{2y^2}{\pi} \int_{-b}^a \frac{p(s)(x-s) ds}{((x-s)^2 + y^2)^2}$$

$$u_x(x, 0) = -\text{sign}(x) \frac{(1-2\nu)(1+\nu)}{2E} \left[ \int_{-b}^x p(s) ds - \int_x^a p(s) ds \right] + C_1$$

$$u_y(x,0) = -\frac{2(1-\nu^2)}{\pi E} \int_{-b}^a p(s) \ln|x-s| ds + C_2$$

## Code

You are provided with two simple Python scripts for evaluation of above-mentioned integrals, however, you'll need to substitute correct expression for the integrands.

- To evaluate stresses  
`compute_stresses_virgin.py`
- To evaluate displacements  
`compute_displacements_virgin.py`

## Objectives

1. Find the stress field in a region which extends to  $x \in [-2a : 2a]$  and  $y \in [-4a : 0]$  for all three pressure distributions mentioned above.
2. Using Hertz's contact formulae, analyze 2D contact between a flat steel substrate ( $E = 210$  GPa,  $\nu = 0.3$ ) and a diamond "cylindrical" indenter of radius  $R = 10$  mm. Determine the point of the onset of plasticity and the corresponding force  $F_c$  using von Mises yield criterion for the yield stress  $R = 350$  MPa. The von Mises stress is given by

$$\sigma_{vM} = \sqrt{\frac{3}{2} \underline{\underline{s}} : \underline{\underline{s}}}$$

where the deviator of the stress tensor is given by

$$\underline{\underline{s}} = \underline{\underline{\sigma}} - \frac{1}{3} \text{tr}(\underline{\underline{\sigma}}) \underline{\underline{I}}.$$

Alternatively, the von Mises stress could be found as:

$$\sigma_{vM} = \sqrt{\frac{1}{2} \left[ (\sigma_x - \sigma_y)^2 + (\sigma_x - \sigma_z)^2 + (\sigma_y - \sigma_z)^2 + 6(\sigma_{xy}^2 + \sigma_{yz}^2 + \sigma_{xz}^2) \right]}$$

*Hint:* don't forget about  $\sigma_{zz}$ .

3. Following a similar procedure, find displacements on the top surface  $u_x(x,0)$ ,  $u_y(x,0)$  for all three pressure distributions mentioned above.