

# Contact Mechanics and Elements of Tribology

## *Foreword*

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# Outline

- Acquaintance
- Questionnaire
- Course content
- Historical remark
- Complexity
- Notations

- Please fill in the questionnaire

## Message to DMS students :

- Rank all suggested problems in order of decreasing attractiveness
- Before the lunch one problem will be assigned to everybody
- On Friday (27/01) you will have 5 minutes (+3 minutes of questions/comments) to present briefly your project to other students, i.e. introduction to the topic (from engineering prospective) and your finite-element model (without results)
- By Friday (3/02/2017) you will need to send me your report with a consistent introduction and solved problem ( $\approx$  10 pages in Français/English)
- Criteria of evaluation : no plagiarism, scientifically sound report

## Lectures :

- 1 Motivation : industrial applications
- 2 Continuum contact mechanics
- 3 Contact mechanics and material behavior
- 4 Computational Contact Mechanics
- 5 Micromechanical contact : roughness
- 6 Micromechanical contact : mechanics
- 7 Contact rheology & friction laws
- 8 Elements of tribology : wear and fretting
- 9 *DMS presentations* + seminars (3 lectures)

# Course content

## Lectures :

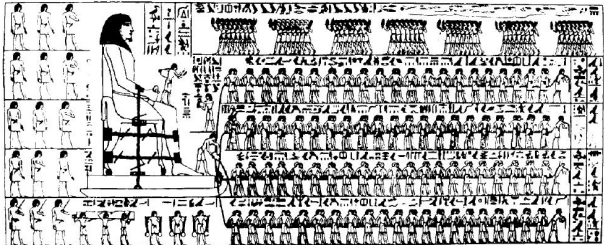
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## Practice :

- 1 Chalk and blackboard : Flamant (Boussinesq, Cerruti)
- 2 Computer : numerical integration of Flamant solution
- 3 Computer : Hertzian contact
- 4 Computer : frictional parabolic contact
- 5 Computer : elasto-plastic contact

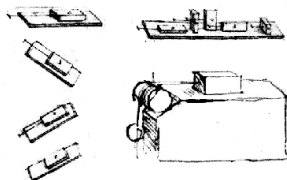
## Use of friction and overcoming of friction

- Frictional heat - lighting of fire > 40 000 years ago.
- Ancient Egypt - lubrication of surfaces with oil > 5 000 years ago.

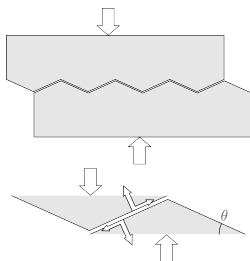


## First scientific studies on contact and friction

- **Leonardo da Vinci** [1452-1519]  
first friction laws and many other tribological topics
- **Isaac Newton** [1687]  
Newton's third law for bodies interaction
- **Guillaume Amontons** [1699]  
rediscovered friction laws
- **Leonhard Euler** [1707-1783]  
roughness theory of friction



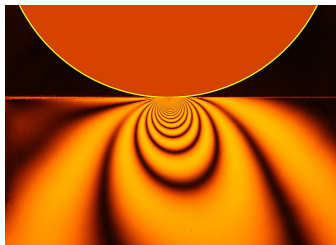
From Leonardo da Vinci's notebook



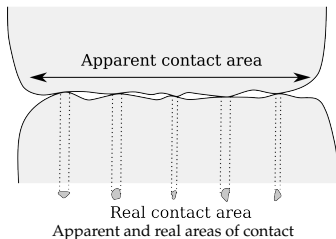
Roughness theory of friction

## First scientific studies on contact and friction

- **Charles-Augustin de Coulomb** [1789]  
friction independence on sliding velocity and roughness, the influence of the time of repose
- **Heinrich Hertz** [1881-1882]  
the first study on contact of deformable solids
- **Holm** [1938],  
**Ernst and Merchant** [1940],  
**Bowden and Tabor** [1942]  
difference between apparent and real contact areas, adhesion theory.

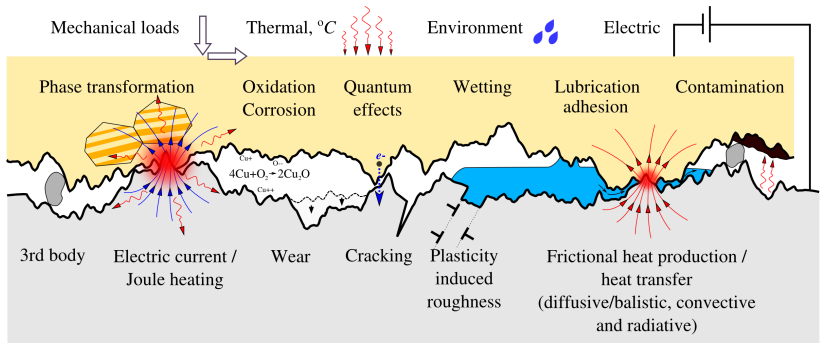


Photoelasticity analysis of Hertz contact problem (fringes correspond to max shear stresses)





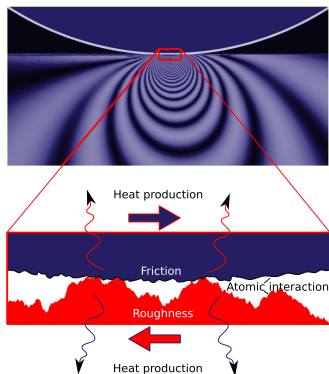
- **Zoom on the contact interface**
- Between different heterogeneous materials with specific rough surfaces
- Under various loads



- **Primary phenomena :** contact, atomic friction, adhesion
- **Secondary phenomena :** macro friction, wear, lubrication, heat production, transport

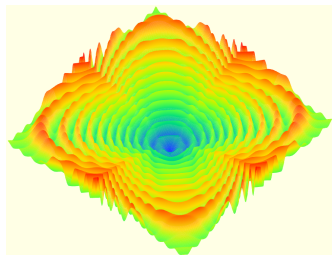
## Particular difficulties related to contact problems : **multiphysical aspects**, mathematical aspects

- Fractality of surfaces
- Interface chemistry
- Hardly accessible contact interface for measurements
- Generation and diffusion of heat
- Multiscale and multiphysical nature of friction



## Particular difficulties related to contact problems : multiphysical aspects, **mathematical aspects**

- One of the most hard problems in mechanics
- Lack of standard optimization problem
- Non-convexity and non-differentiability
- Non-continuous character
- Bad scalability



## Vectors and tensors

- $a, \alpha$  scalars
- $\underline{b}$  vectors
- $\underline{\underline{C}}, \underline{\underline{\beta}}$  2nd order tensors
- $\underline{\underline{\underline{D}}}$  4th order tensors
- $\nabla \underline{a} = \underline{\underline{B}}$  gradient operator
- $\nabla \times \underline{a} = \underline{\underline{B}}$  rotor operator
- $\underline{a} \cdot \underline{b} = c$  scalar (dot) product
- $\underline{a} \times \underline{b} = \underline{c}$  vector (cross) product
- $\underline{a} \otimes \underline{b} = \underline{\underline{C}}$  tensor product
- $\underline{\underline{A}}^T$  transposition
- $\nabla \cdot \underline{a} = c$  divergence operator
- $\underline{\underline{I}} = \underline{e}_i \otimes \underline{e}_i$  unitary 2nd order tensor

## Mechanics

- $\underline{\underline{\sigma}}$  Cauchy stress tensor
- $g, g_n$  gap, normal gap
- $\epsilon$  penalty parameter
- $\lambda, \lambda_n, \lambda_t$  lagrange multipliers
- $\sigma_n = (\underline{\underline{\sigma}} \cdot \underline{n}) \cdot \underline{n}$  contact pressure
- $\underline{\underline{\epsilon}}$  Small strain tensor
- $\underline{\underline{\xi}}$  position vector in parent space
- $\underline{n}$  outward unit normal vector
- $\frac{\partial \rho}{\partial \xi_1}, \frac{\partial \rho}{\partial \xi_2}$  surface tangent vectors
- $f, \mu$  Coefficient of friction



Welcome to the CMET course!

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