Computational Approach to Micromechanical Contacts

Lecture 4.b

Computational Contact Mechanics (BEM)

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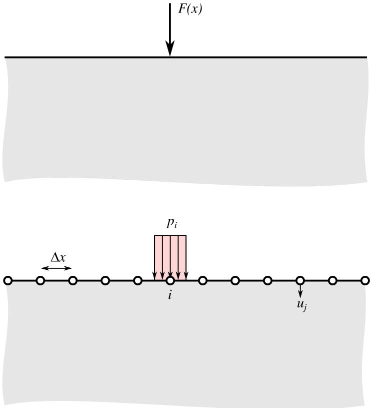
Main idea

- Recall Flamant's or Boussinesq's solution for a single force acting on a half-space
- Due to pressure p distributed in $x \in [a, b]$, ertical displacement u on any location x is given by

$$u(x) = u_0 + \frac{2(1-\nu^2)}{\pi E} \int_a^b p(s) \log|s-x|\, ds$$

- Discretize the surface traction into piece-wise constant values defined over a regularly spaced grid: p_i at x_i with spacind Δx
- Then the displacement at location x_j due to a single pressure element located at x_i is

$$u_j = u_0 + \frac{2(1-\nu^2)}{\pi E} \left(\int_{x_i-\Delta x/2}^{x_i+\Delta x/2} \log|s-x_j|\, ds \right) p_i$$



Continuous and discretized formulations

Main idea II

- Or simply

  $u_j = u_0 + S_{ji} p_i$

  where $S_{ji} = \dfrac{2(1-\nu^2)}{\pi E} \displaystyle\int\limits_{x_i-\Delta x/2}^{x_i+\Delta x/2} \log|s - x_j|\, ds$

- Define a rough surface s_i, the gap is given

  $g_i = s_i - u_i$

- Find p_i such that

  $\forall x_i : p_i \geq 0, \quad g_i \geq 0, \quad g_i p_i = 0$

- **Mixed BC formulation:**

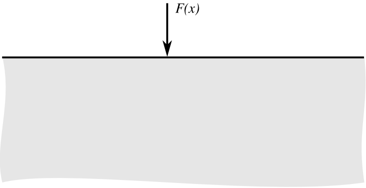
  in contact $i, j \in C$:

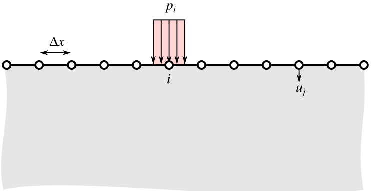
  $\tilde{S}_{ji} \tilde{p}_i = \tilde{s}_j - u_0, \quad p_i \geq 0, \quad \Delta x \sum p_i = F_{tot}$

  where $\tilde{S}_{ji}$ is a submatrix of S_{ji}

  out of contact $i, j \in \mathcal{NC}$:

  $S_{ji} p_i \leq s_j - u_0, \quad p_i = 0$





Continuous and discretized formulations

Computational approach

- Prescribe indeter's surface $s(x) \to s_i$
- Prescribe indentation depth u_0
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  3. Complete the pressure vector

     $p_i = 0 \quad \text{if} \quad \forall i \notin C$
  4. Construct full solution

     $S_{ji}p_i = u_j - u_0$
  5. If $u_j - u_0 > s_j$ add j in C
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  7. If C no longer changes, then convergence is reached.

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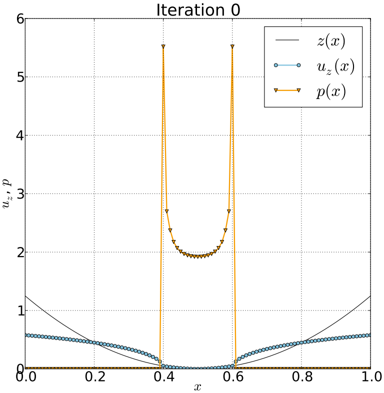

Example: cylindrical indenter

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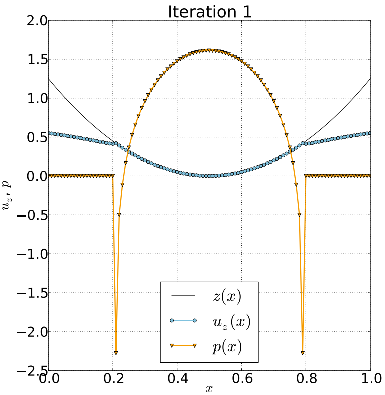

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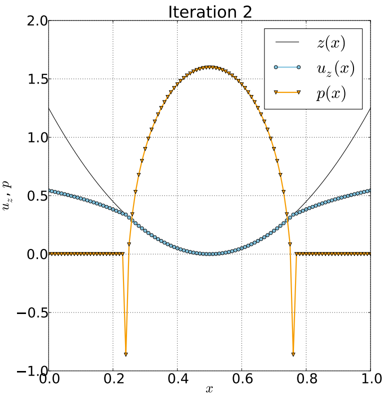

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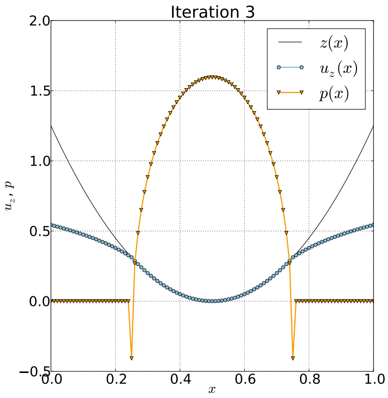

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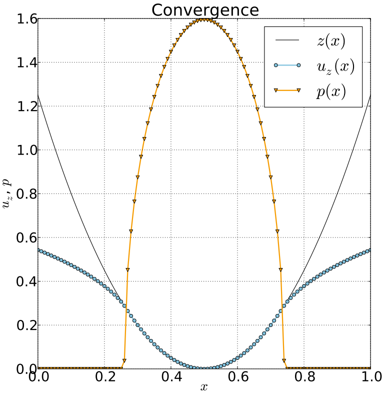

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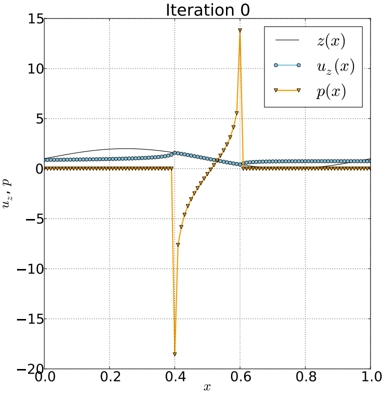

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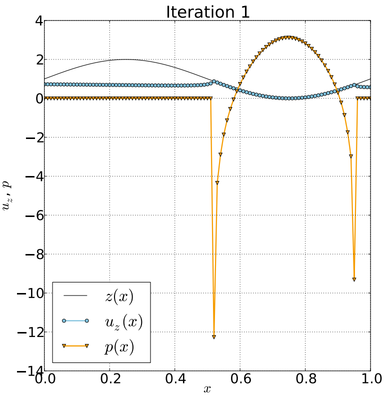

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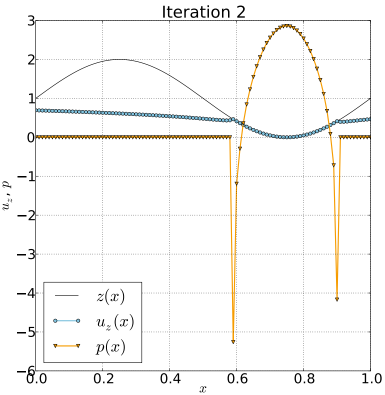

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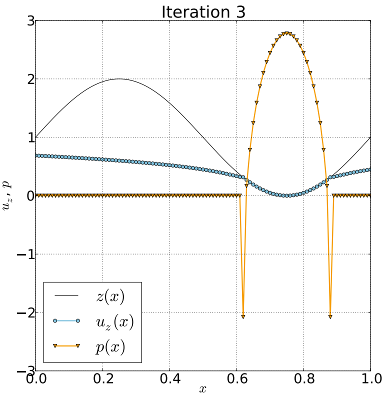

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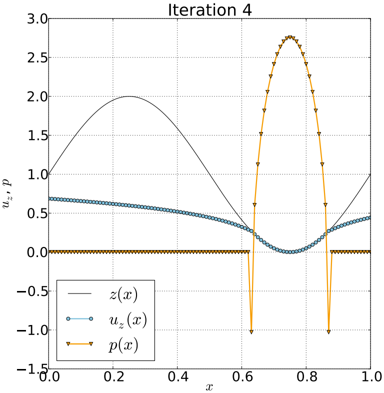

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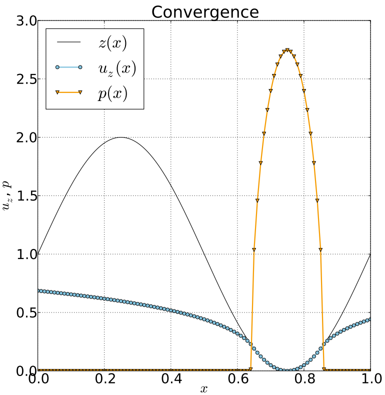

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Spectral approach

- Relationship between vertical displacement and pressure

$$p(x,y) = p_0 \cos(k_x x) \cos(k_y y)$$

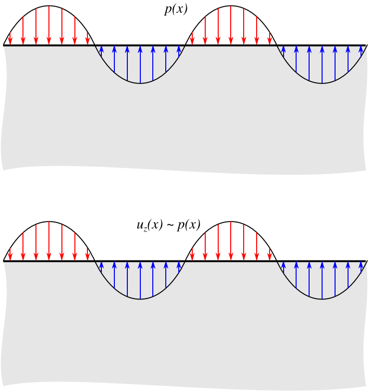
$$u_z(x,y) = \frac{p_0}{E^* \sqrt{k_x^2 + k_y^2}} \cos(k_x x) \cos(k_y y)$$

- Then passing throug Fourier space $\mathcal{F}$:

$$u_z = \mathcal{F}^{-1}\left\{w : \mathcal{F}(p)\right\}$$

- Where

$$w_{kl} = \frac{L}{\pi E^* \sqrt{k^2 + l^2}}$$



Pressure/displacement proportionality