Computational Approach to Micromechanical Contacts

Lecture 7.

Elastodynamic friction between dissimilar materials

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Applications

Relevant applications
& phenomena

1 Interface cracks in mixed mode

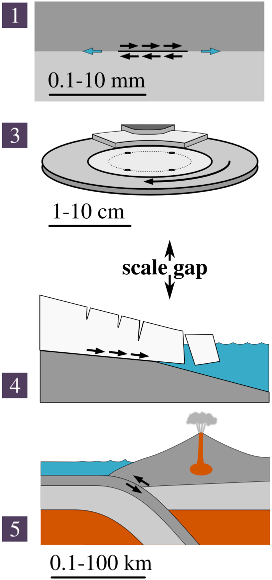
2 Touch interfaces

3 Brake systems

scale gap

4 Glacier basal slip

5 Slip in faults



Relevant applications & phenomena

1. Interface cracks in mixed mode
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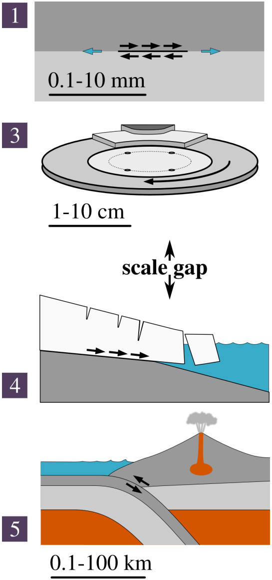
3. Brake systems

   scale gap

4. Glacier basal slip
5. Slip in faults

Scalable phenomena

1. Earthquakes can be reproduced in laboratory



Physics of frictional sliding

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Coulomb-Amontons law

$$\boxed{\begin{cases} |\tau| < f|\sigma|, & |\dot{u}| = 0 \\ |\tau| = f|\sigma|, & |\dot{u}| > 0 \end{cases}}$$

τ - tangential traction,
σ - contact pressure,
$\dot{u}$ - slip velocity,
f - coefficient of friction.

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  - scale
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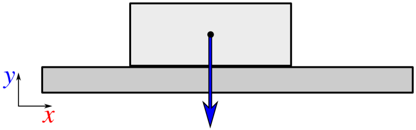
- Local vs global friction



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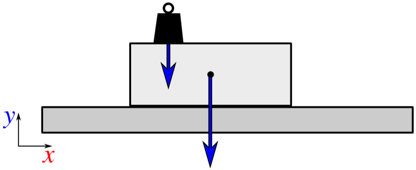

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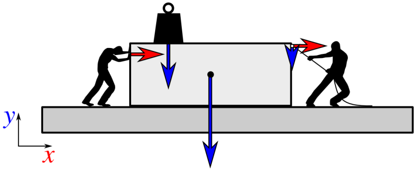

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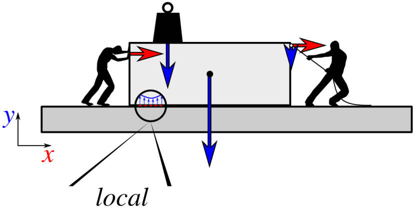

τ, σ

$$f_{loc} = \max(|\tau / \sigma|)$$

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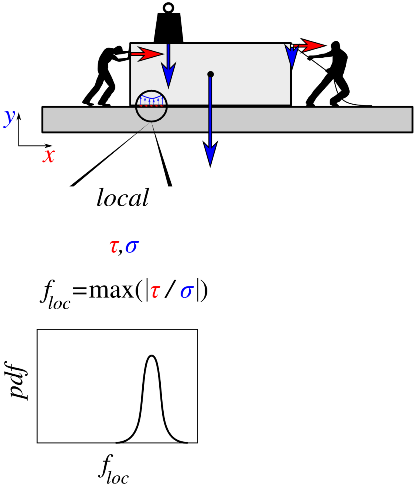

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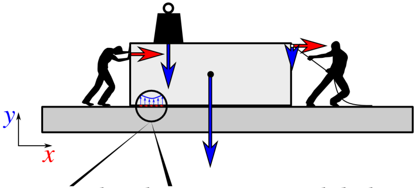
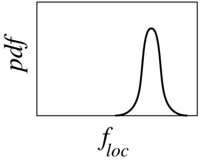


local   *global*

τ, σ $T = \Sigma F_x,\ N = \Sigma F_y$

$f_{loc} = \max(|\tau / \sigma|)$ $f_{gl} = \max(|T / N|)$



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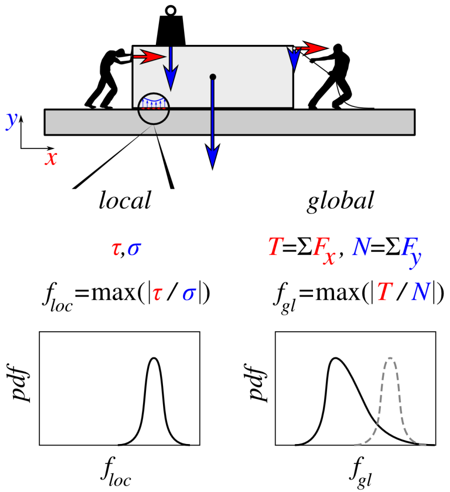

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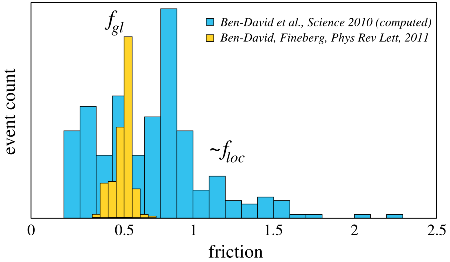

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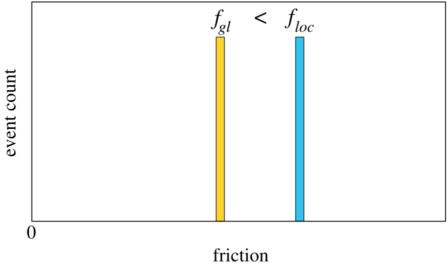

(Weertman, 1980) Unstable slippage across a fault that separates elastic media of different elastic constants, *J Geo Research*

(Adams, 1995) Self-excited oscillations of two elastic half-spaces sliding with a constant coefficient of friction, *J Appl Mech*

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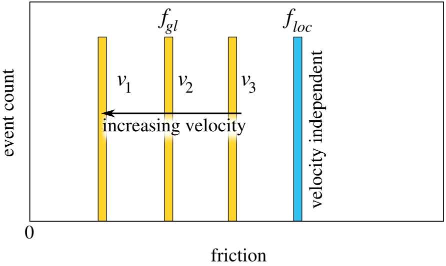

(Adams, 2000) Radiation of body waves induced by the sliding of an elastic half-space against a rigid surface, *J Appl Mech*

(Moirot, Nguyen, Oueslati, 2002) An example of stick-slip and stick-slip-separation waves, *Europ J Mech A Solids*

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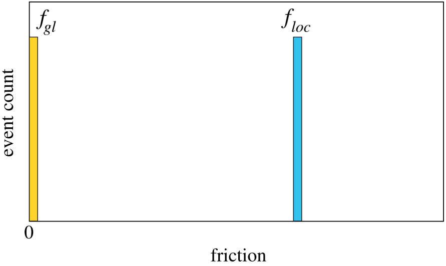

(Cochard & Rice, 2001) Fault rupture between dissimilar materials: Ill-posedness, regularization, and slip-pulse response, J Geophys Res

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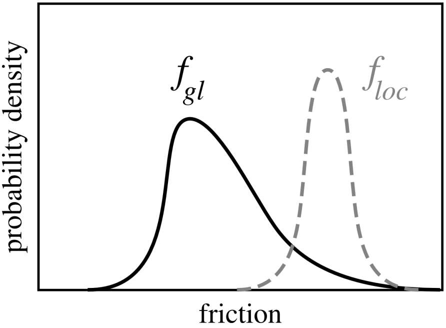
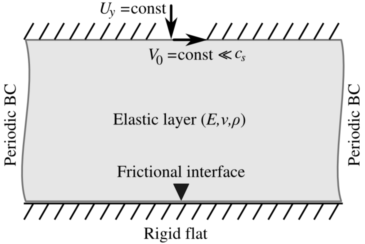


- What are the relevant local friction laws?
- How does the dynamics affect the frictional slip?
- How can it be used for practice?

Problem set-up

- **Problem:** Elastic layer sliding on a rigid flat under Coulomb friction.



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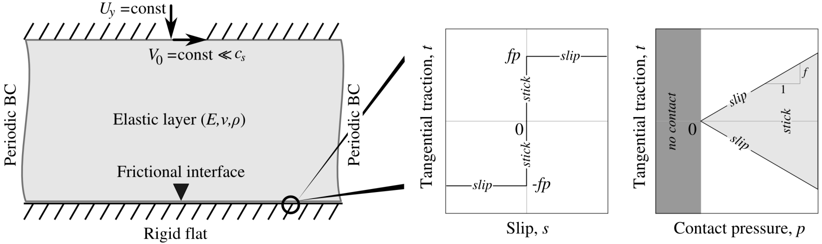

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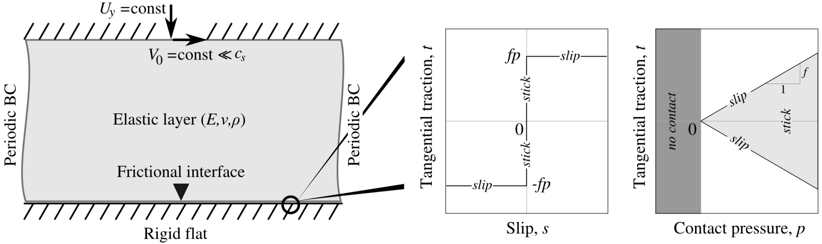

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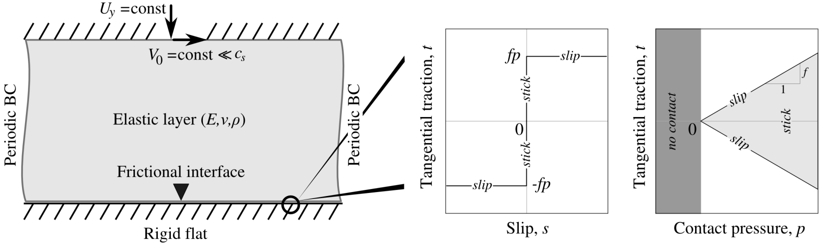


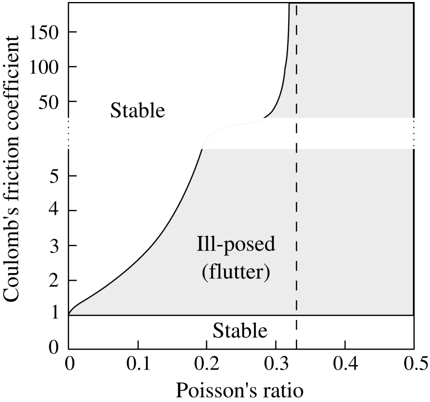
- **Parameters:** Poisson's ratio ν, friction f and sliding velocity V_0.
- **Methods:** Implicit dynamic finite element simulation, α-method HHT[1], direct method to solve frictional contact[2,3].
  FE mesh: 33 000 quadrilateral elements with reduced integration.

[1] **(Hilber, Hughes and Taylor, 1977)** Improved numerical dissipation for time integration algorithms in structural dynamics, *Earthq Eng Struct D*

[2] **(Francavilla & Zienkiewicz, 1975)** A note on numerical computation of elastic contact problems, Int J Num Meth Eng.

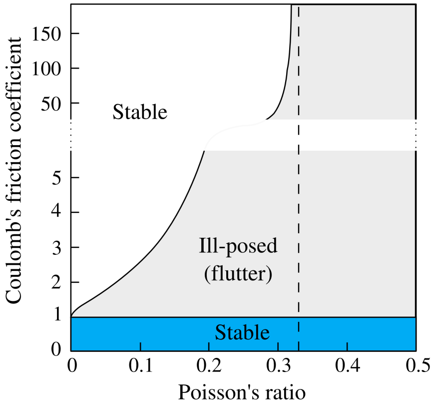
[3] **(Jean, 1995)** Frictional contact in collections of rigid or deformable bodies: numerical simulation of geomaterial motions, *Stud Appl Mech*

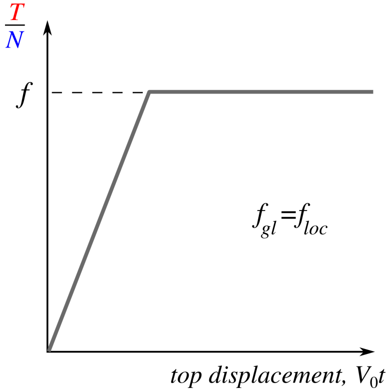
Limitations and expectations



(Renardy, 1992 [1989]) Ill-posedness at the boundary for elastic solids sliding under Coulomb friction, *J Elast*

(Martins, Guimarães & Faria, 1995 [1993]) Dynamic Surface Solutions in Linear Elasticity and Viscoelasticity With Frictional Boundary Conditions, *J Vibr Acoust*





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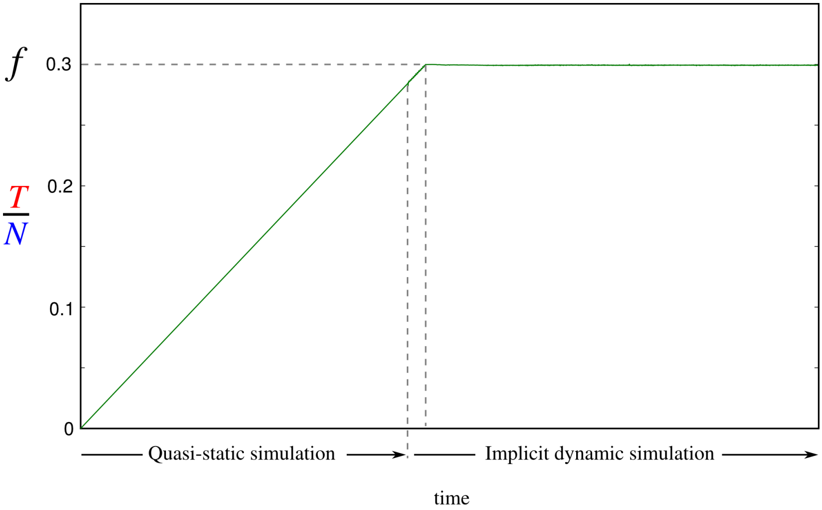
Results I: $f = 0.3$

Parameters: $\nu = 0.2, f = 0.3, V_0 = 10^{-6} c_s$.



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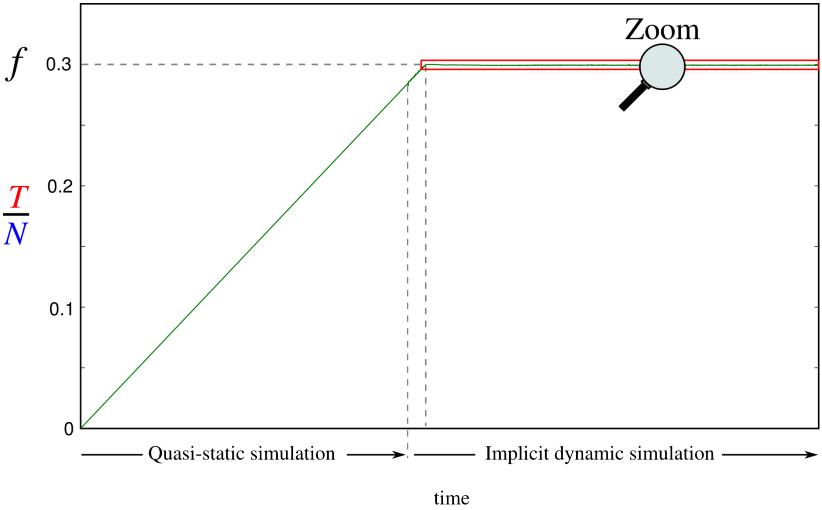

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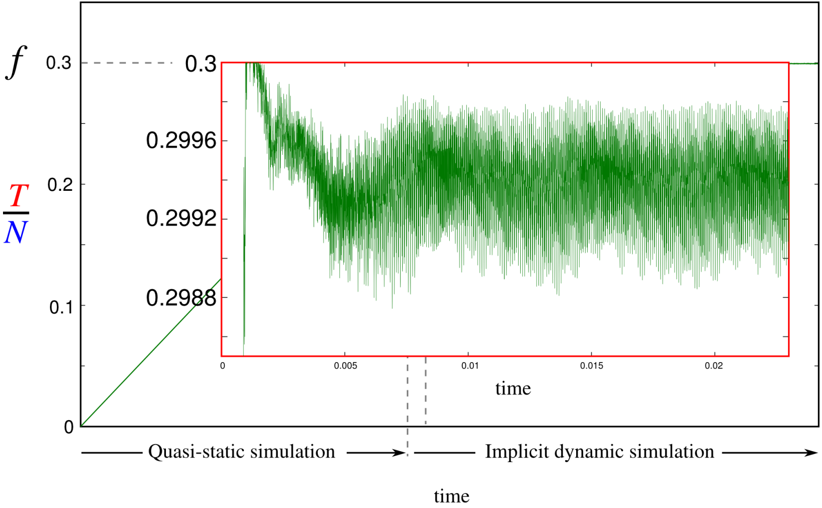

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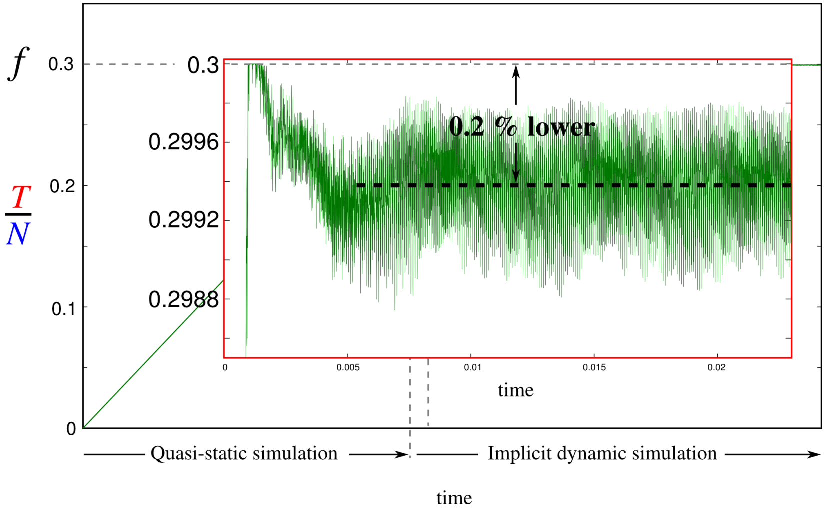

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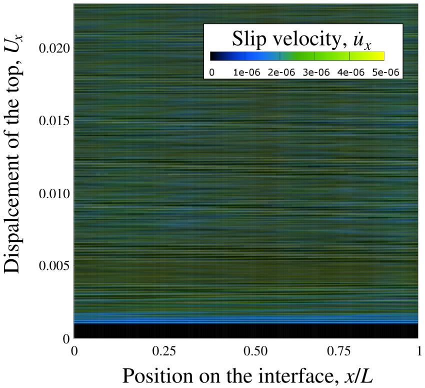

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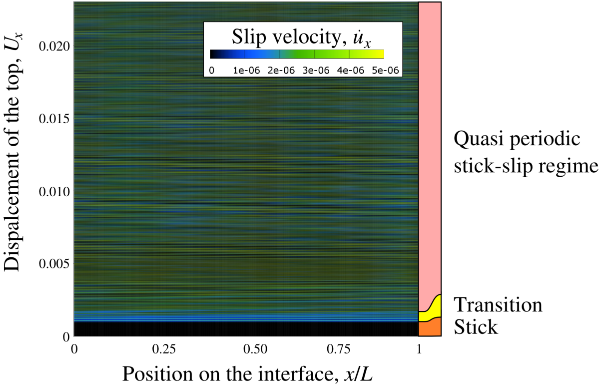

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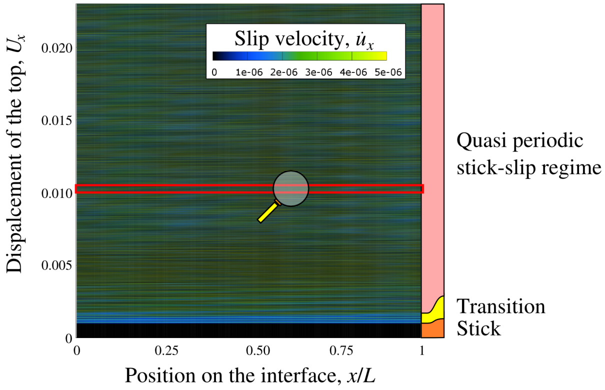

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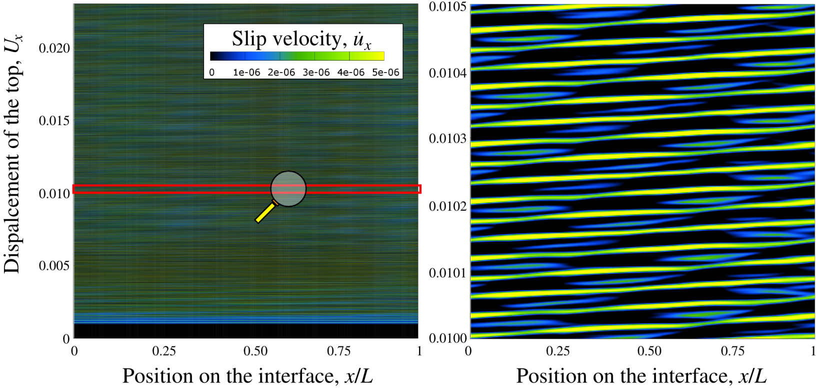

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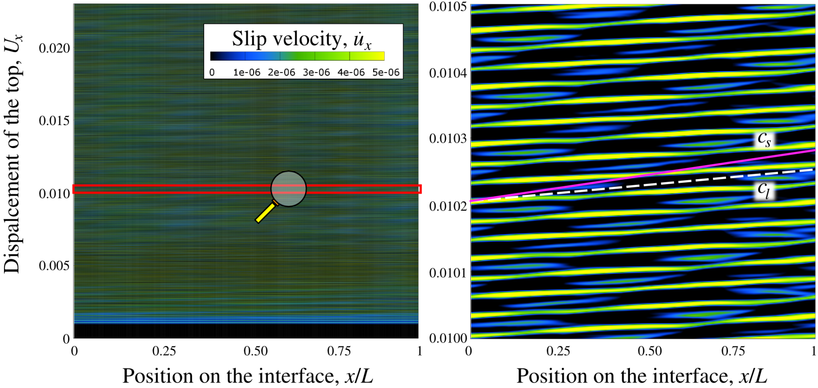

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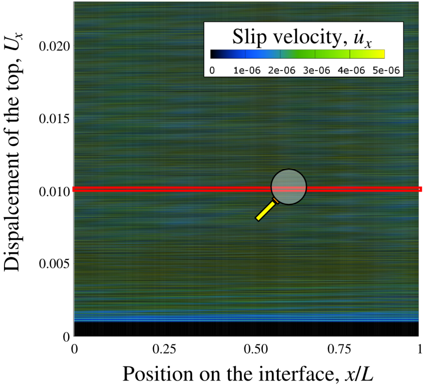

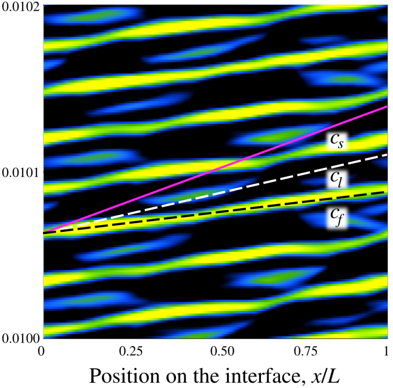

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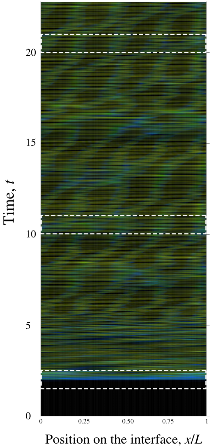

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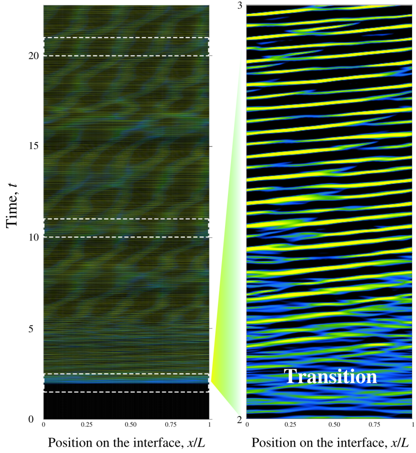

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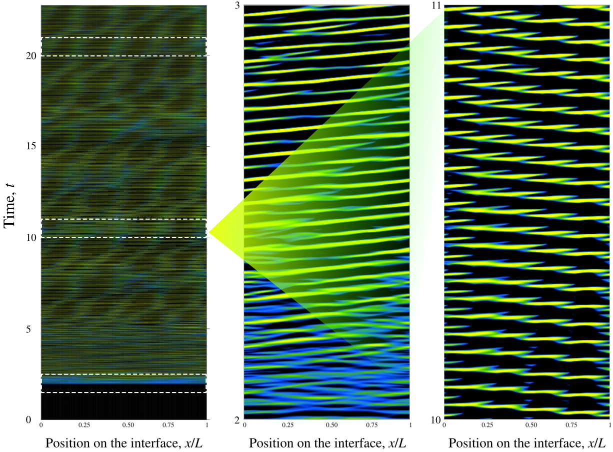

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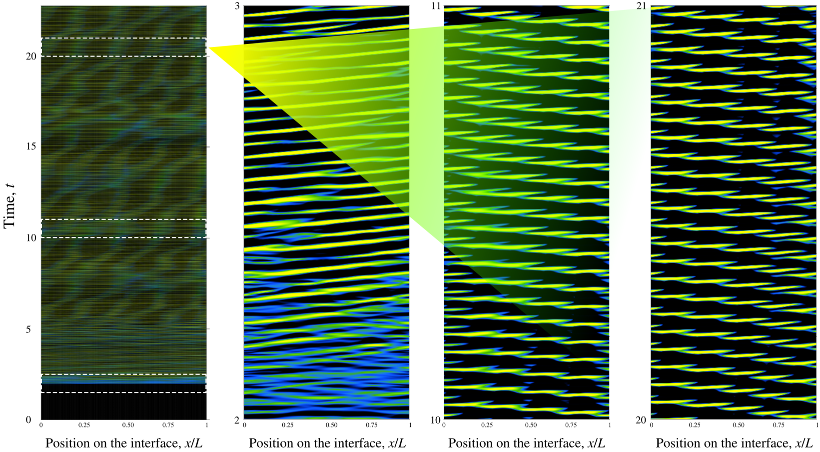

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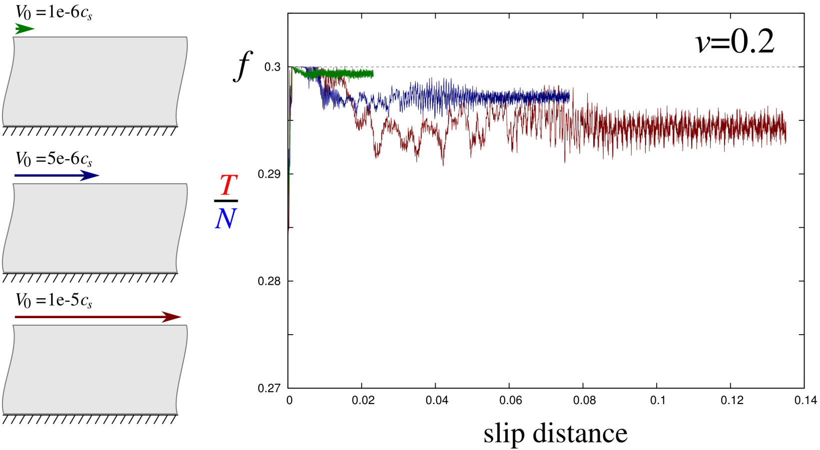

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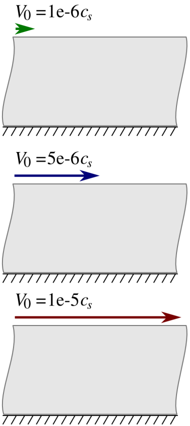

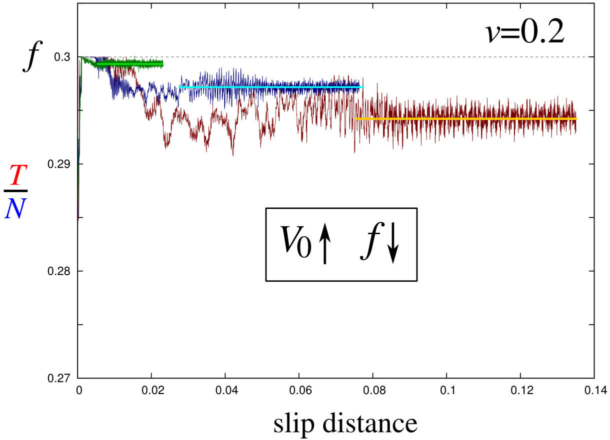

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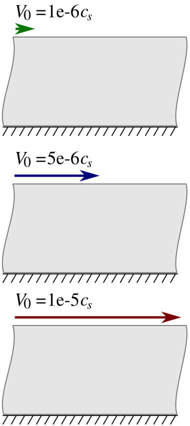

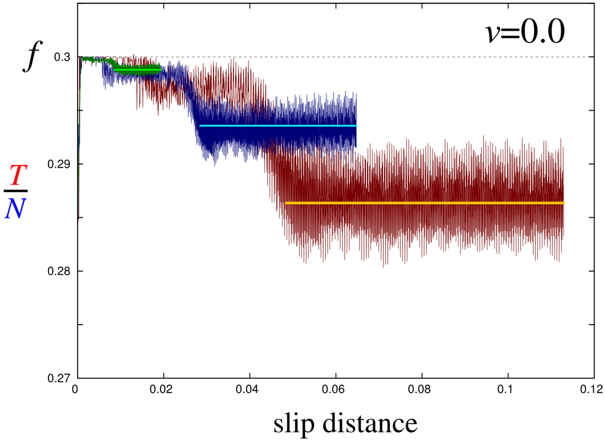

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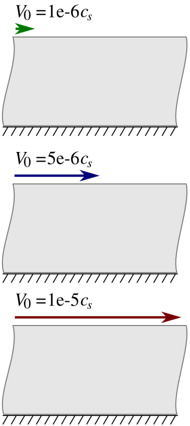

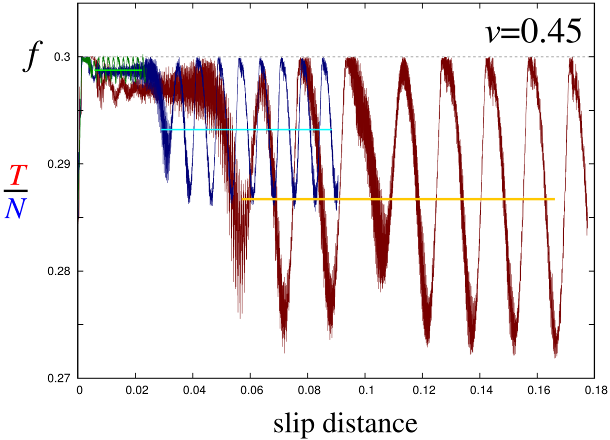

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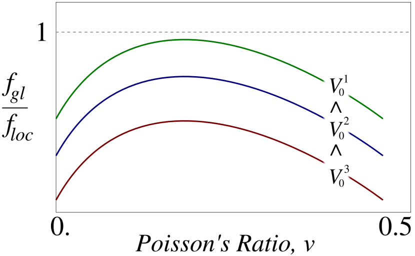


- For a train of rectangular stick-slip waves **(Adams, 2000)** predicted velocity dependence:

$$f_{gl} = f_{loc} - \left(\frac{f}{f_{loc}} - 1\right)\frac{S\alpha V_0 G}{(L-S)\sigma_{yy}c_s},$$

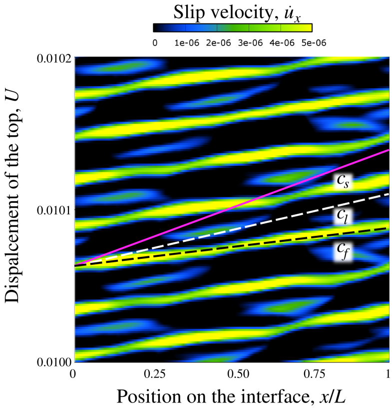
where S is the slip length, L period, G shear modulus, $f = \sigma_{xy}/\sigma_{yy}$ in body waves, $\alpha = \alpha(c_f, c_l, f)$ a coefficient.

(Adams, 2000) Radiation of body waves induced by the sliding of an elastic half-space against a rigid surface, *J Appl Mech*

Intermediate discussion:

- uniform slip in *finite size systems* is unstable
- stick-slip waves
- intersonic $c_s < c_p \le c_l$ and supersonic pulses $c_p > c_l$
- global friction is reduced

$$\boxed{f_{gl} < f_{loc}}$$



Results I: discussion

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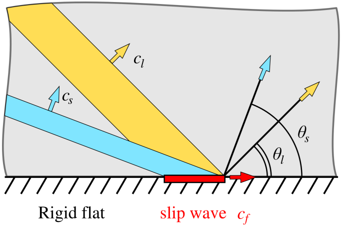
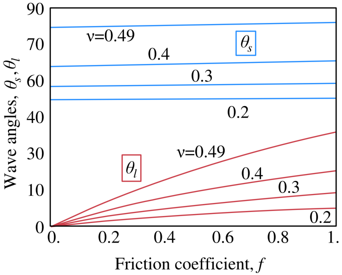
$$\boxed{f_{gl} < f_{loc}}$$

Explanation:

- Waveguide modes[1,2]
- Stick-slip waves[3]
- "Radiation of body waves induced by the sliding"[4]

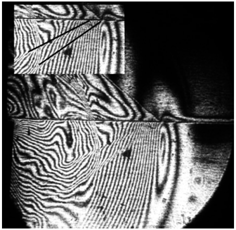




[1] **(Mindlin, 1955)** An introduction to the mathematical theory of vibrations of elastic plates.
[2] **(Brener et al., 2016)** Dynamic instabilities of frictional sliding at a bimaterial interface, J Mech Phys Solids
[3] **(Bui & Oueslati, 2010)** On the stick-slip waves under unilateral contact . . . , *Ann Solid Struct Mech*
[4] **(Adams, 2000)** Radiation of body waves induced by the sliding of an elastic half-space . . . , *J Appl Mech*

Results I: discussion II

Supersonic slip propagation does not violate causality
even though c_l is the maximal signal speed



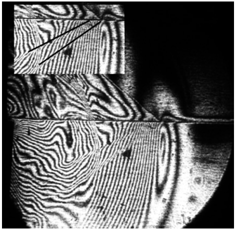
(Coker, Rosakis, Needleman, 2003) Dynamic crack growth along a polymer composite-Homalite interface, *J Mech Phys Solids*

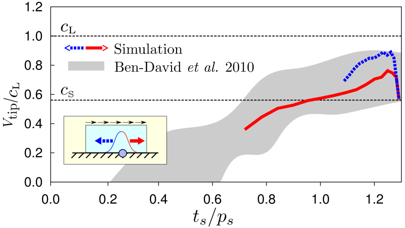
(Coker, Lykotrafitis, Needleman, Rosakis, 2005) Frictional sliding modes along an interface between identical elastic plates subject to shear impact loading, *J Mech Phys Solids*

(Kammer & Yastrebov, 2012) On the Propagation of Slip Fronts at Frictional Interfaces, *Tribol Lett*

Results I: discussion II

Supersonic slip propagation does not violate causality
even though c_l is the maximal signal speed





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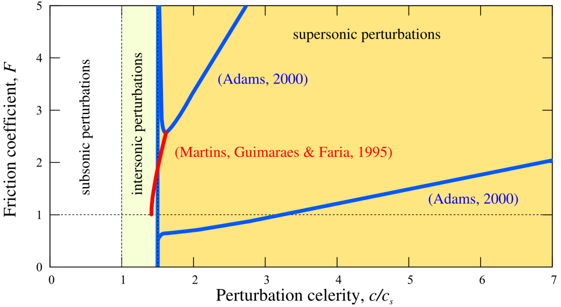
(Kammer & Yastrebov, 2012) On the Propagation of Slip Fronts at Frictional Interfaces, *Tribol Lett*

(Ben-David, Cohen & Fineberg 2010) The dynamics of the onset of frictional slip, *Science*

Results I: discussion III

Inter- and supersonic stable and unstable roots for $\nu = 0.1$



(Martins, Guimarães & Faria, 1995) Dynamic Surface Solutions in Linear Elasticity . . . , *J Vibr Acoust*
(Adams, 2000) Radiation of body waves induced by the sliding of an elastic half-space . . . , *J Appl Mech*

- Neglect supersonic perturbations in half-space

  (Ranjith & Rice, 2001) Slip dynamics at an interface between dissimilar materials, *J Mech Phys Solids*

  (Bui & Oueslati, 2010) On the stick-slip waves under unilateral contact and Coulomb friction, *Ann Solid Struct Mech*

- Neglect supersonic perturbations in an elastic-layer
  (Brener et al., 2016) Dynamic instabilities of frictional sliding at a bimaterial interface, *J Mech Phys Solids*

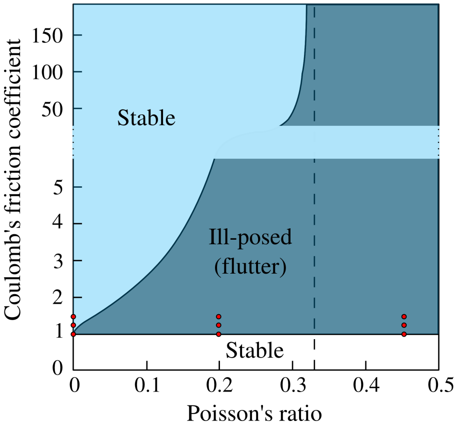
Within the ill-posed region

Preliminary comments:

- Ill-posed[1] if

$$\sigma_{xy} = f\sigma_{yy}$$

  holds all along the interface
- Exponential growth of amplitude results in a stick-slip or separation
- Critical region $f \geq 1$ for high frequencies $kH \gg 1$[2-4]

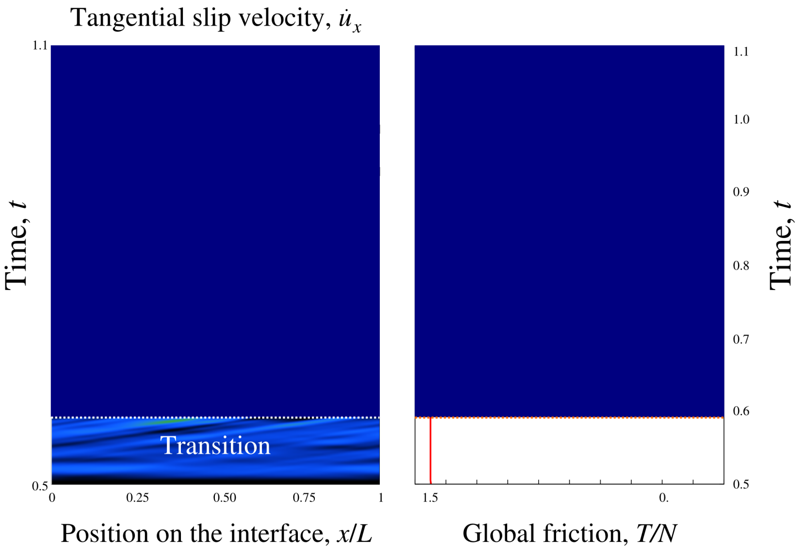


[1] **(Martins, Guimarães & Faria, 1995)** Dynamic Surface Solutions in Linear Elasticity . . . , *J Vibr Acoust*

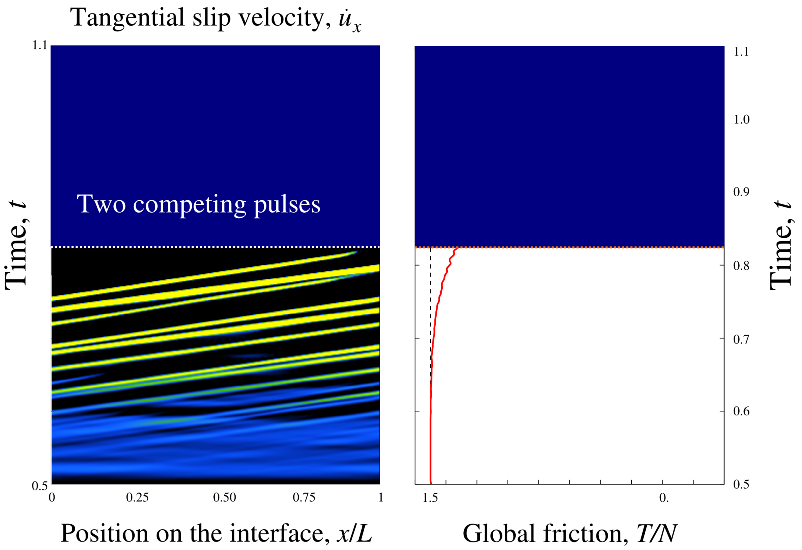
[2] **(Ranjith & Rice, 2001)** Slip dynamics at an interface between dissimilar materials, *J Mech Phys Solids* 49

[3] **(Cochard & Rice, 2000)** Fault rupture between dissimilar materials: Ill-posedness . . . , *J Geophys Res* 105

[4] **(Kammer, Yastrebov, Anciaux, and Molinari, 2014)** The existence of a critical length scale in regularised friction, *J Mech Phys Solids* 63


Tangential slip velocity, $\dot{u}_x$
Time, t
1.1
0.5
0
0.25
0.50
0.75
1
Transition
Position on the interface, x/L
1.5
0.
Global friction, T/N
1.1
1.0
0.9
0.8
0.7
0.6
0.5
Time, t

Results IV: within the ill-posed region $f = 1.5$



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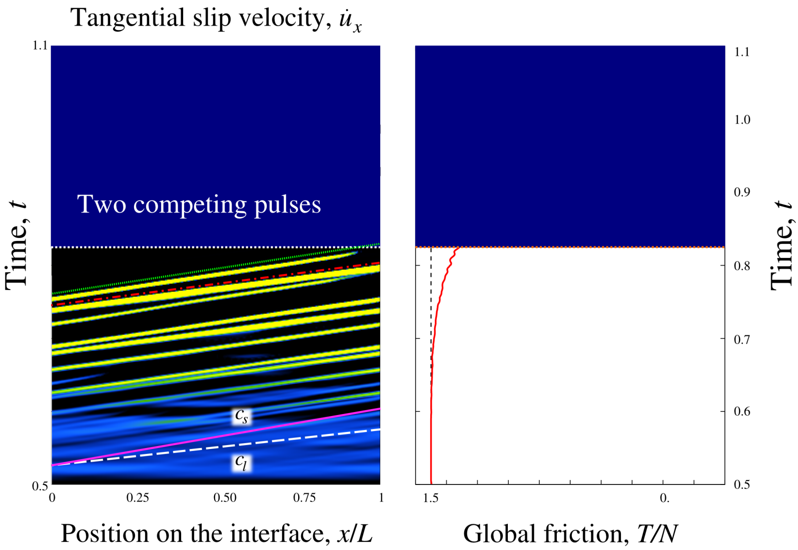

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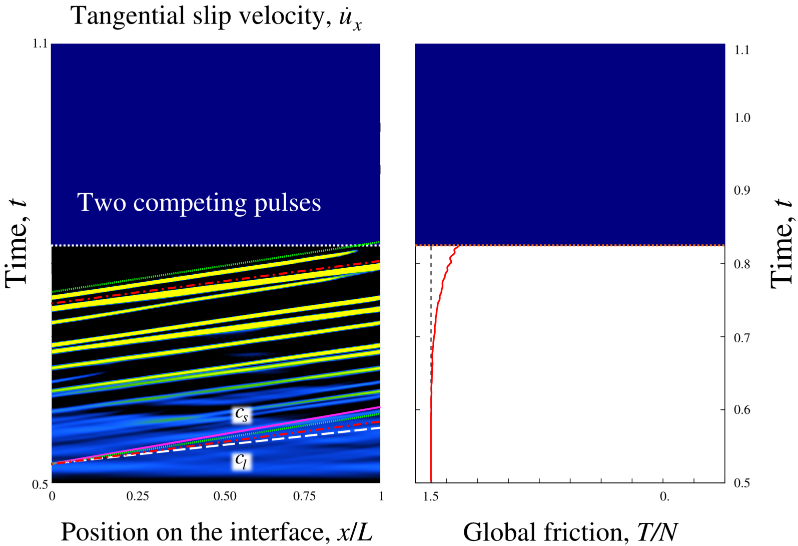

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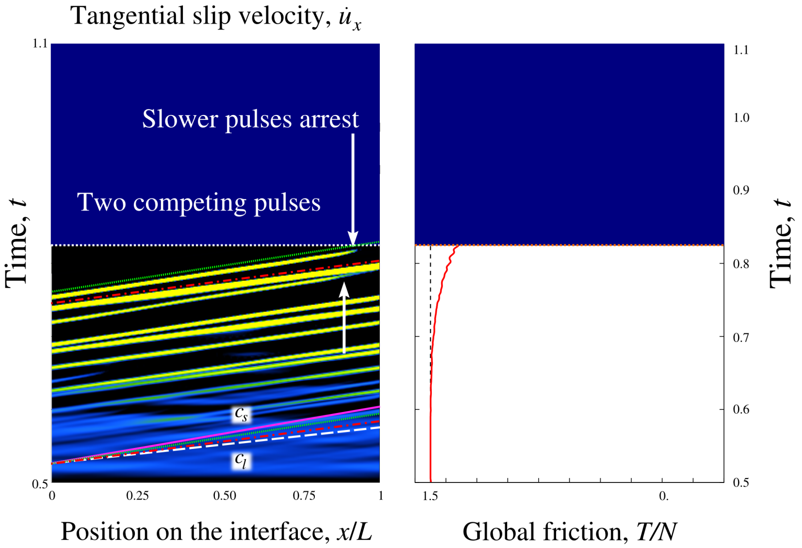

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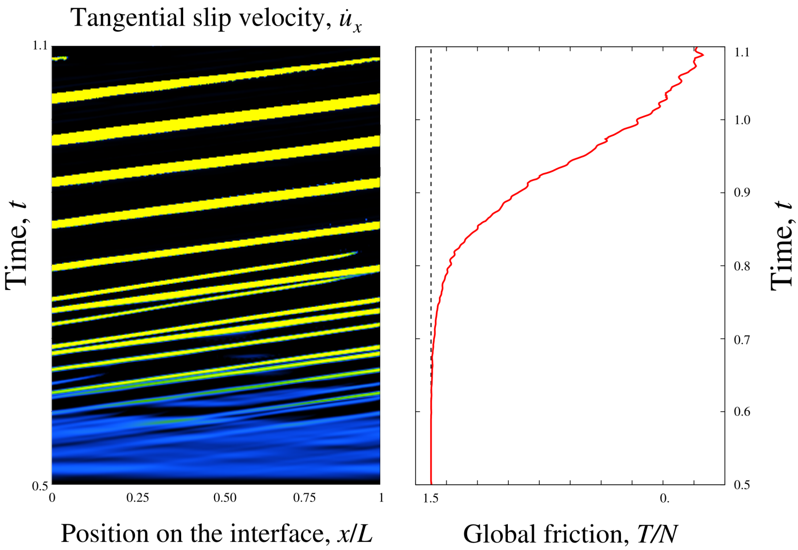

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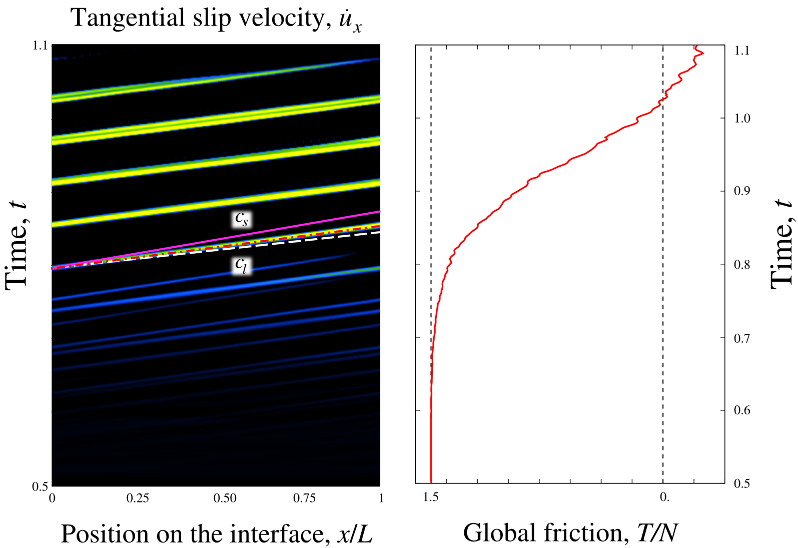

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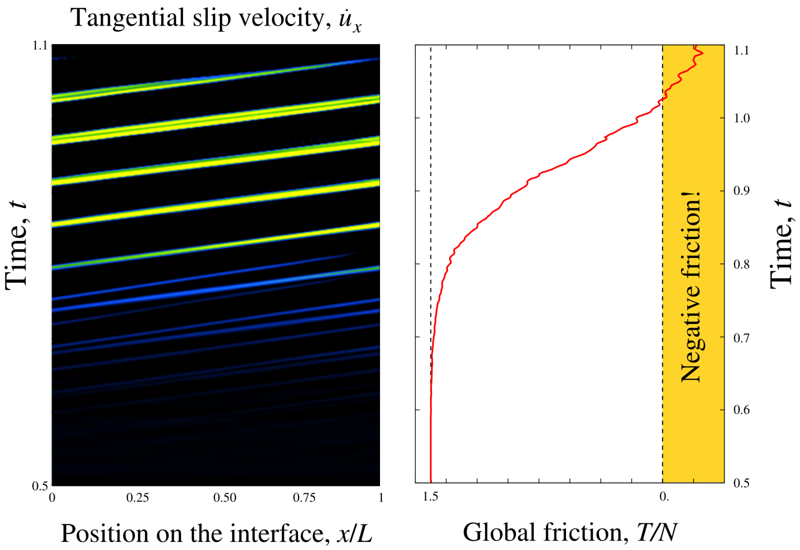

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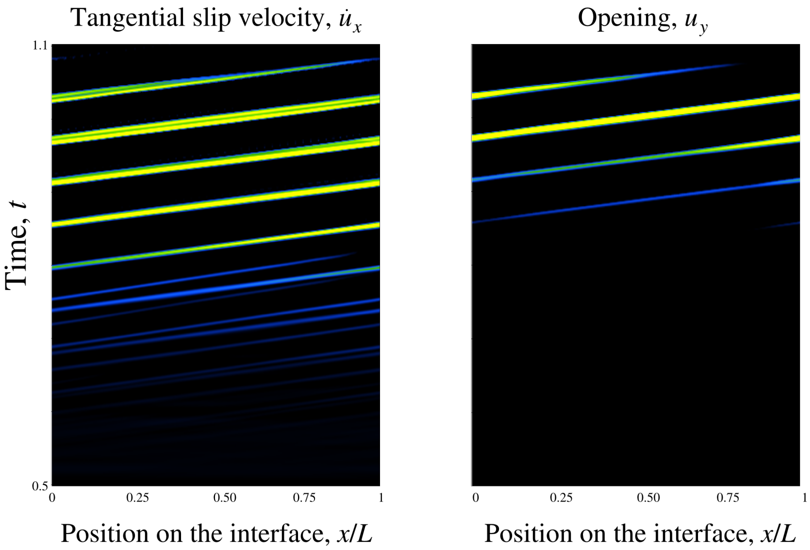

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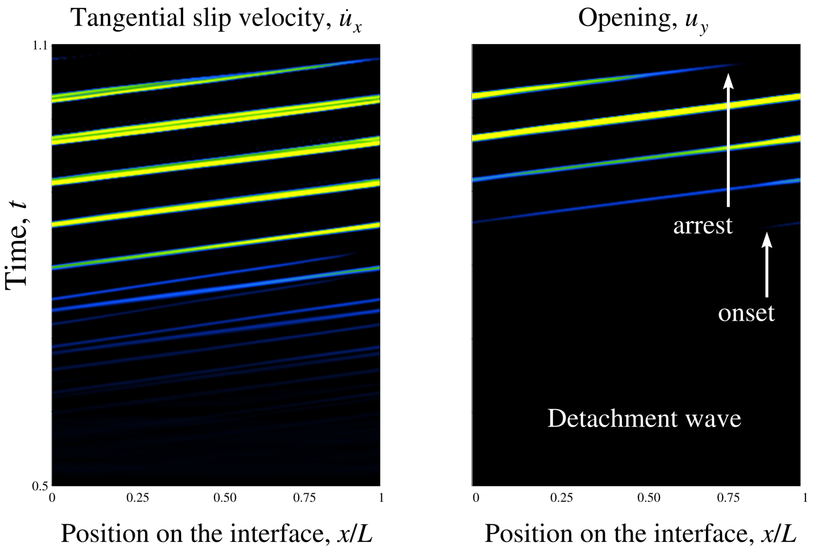

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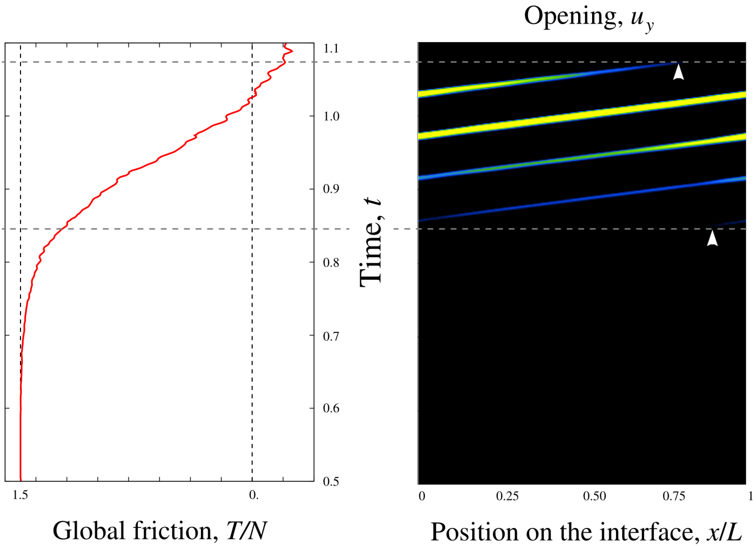
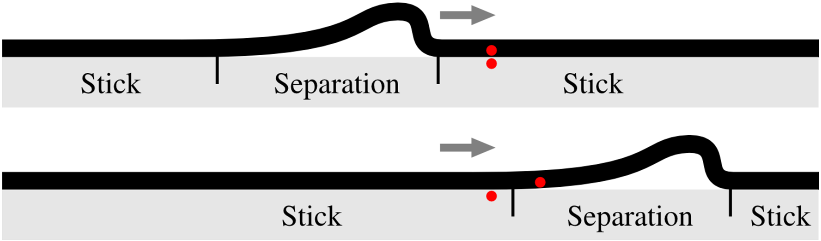


Results IV: animation



Results IV: animation

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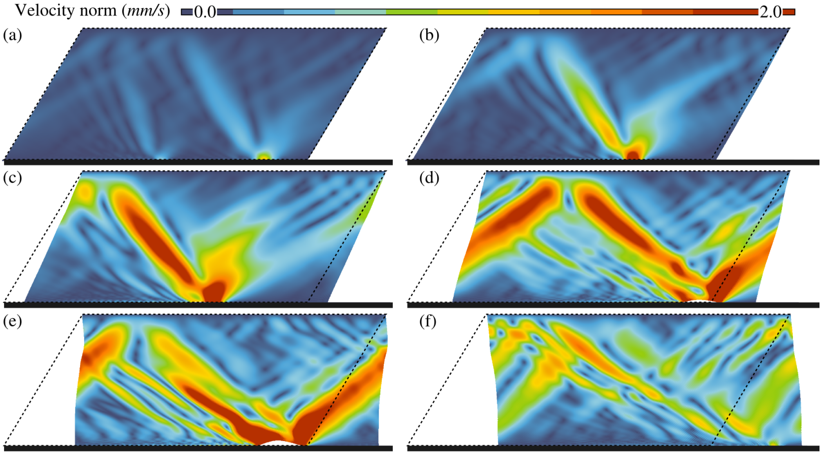
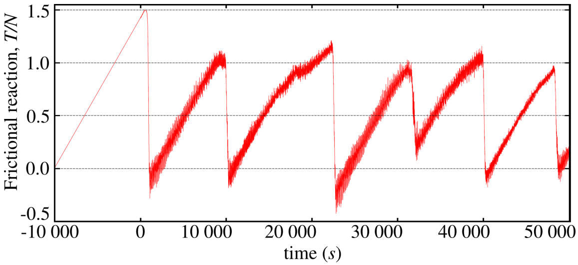
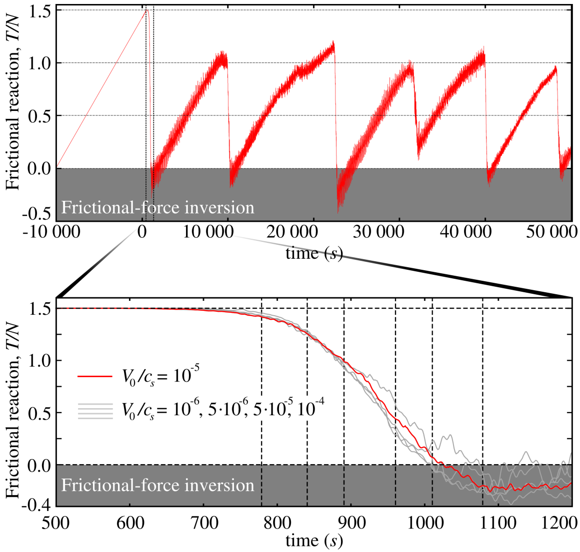


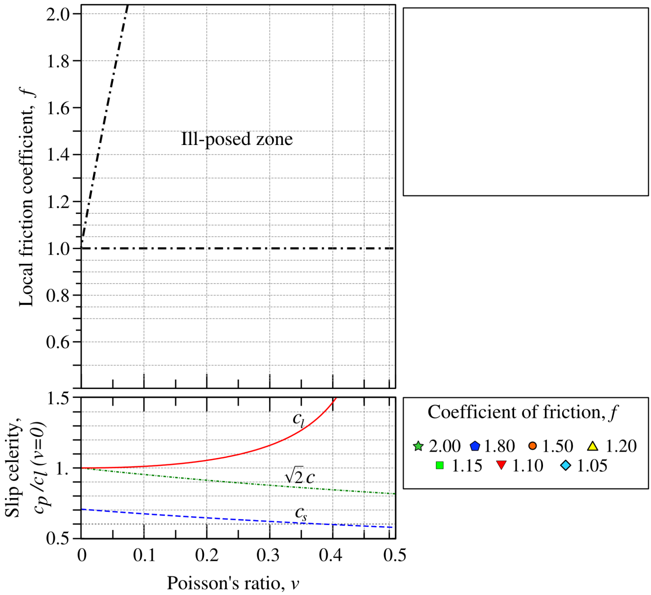
Results IV: behavior at longer periods



Results IV: behavior at longer periods



Results IV: parametric study



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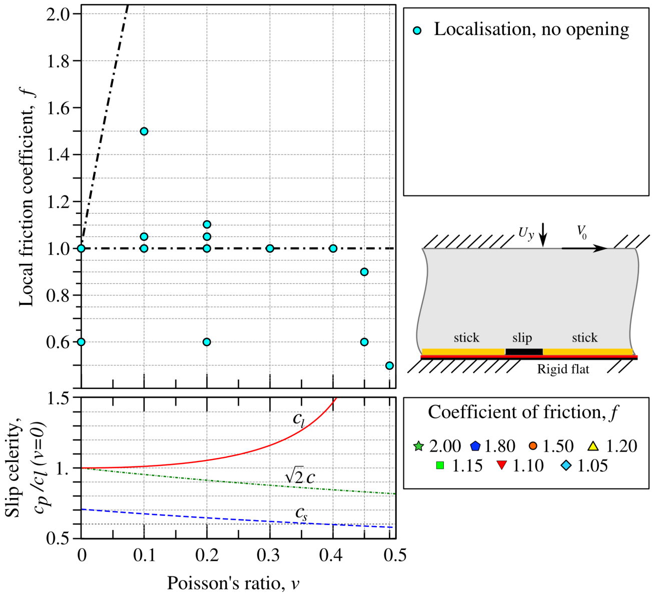

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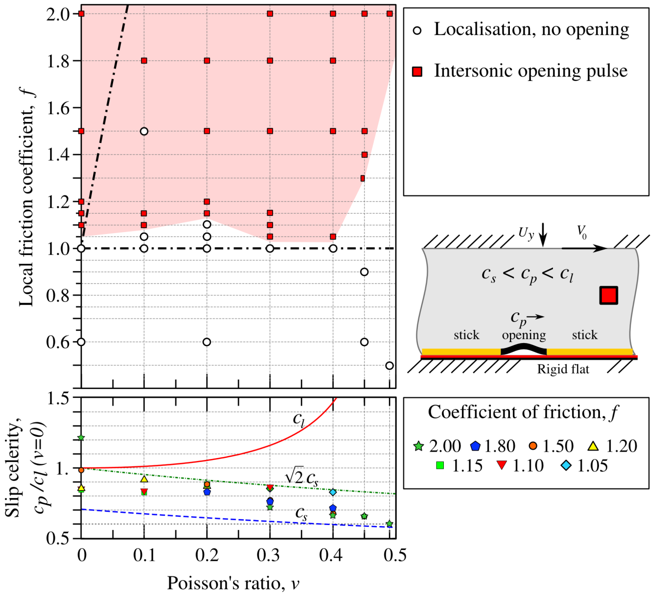
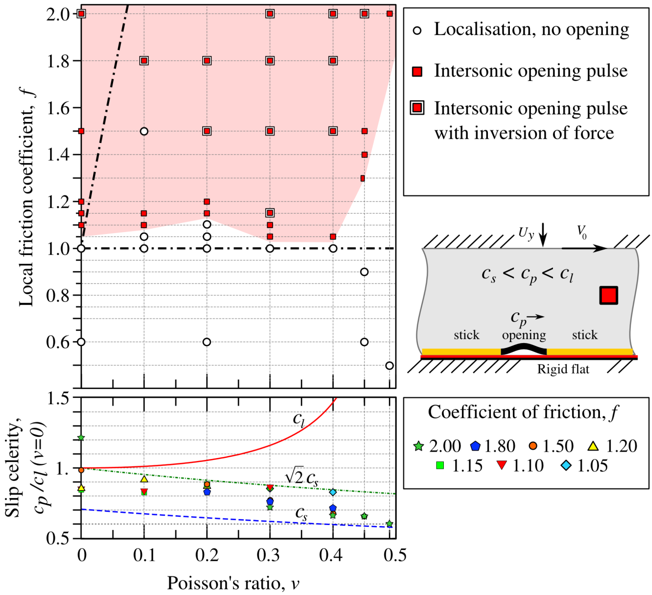


Results IV: parametric study



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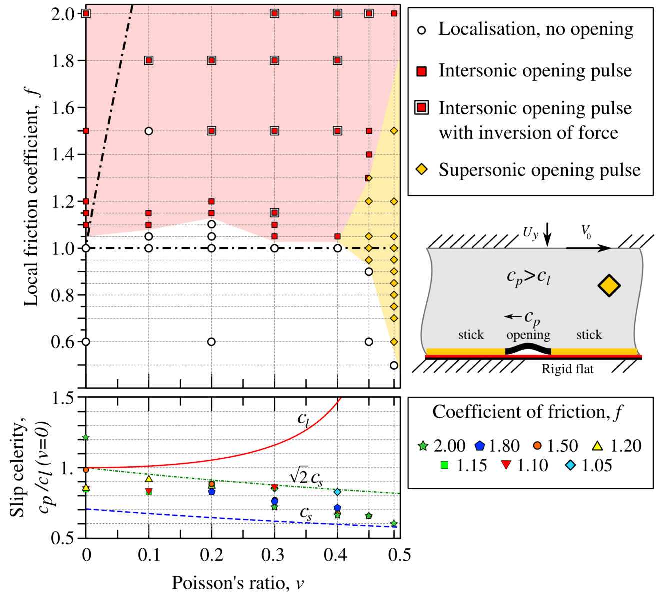


Formal analysis I

- Scalar elastodynamic potentials for dilatational and shear waves[1]:

$$\varphi = f(y)\exp[ik(x-ct)], \quad \psi = g(y)\exp[ik(x-ct)]$$
$$f(y) = A\sin(\eta_d y) + B\cos(\eta_d y), \quad g(y) = C\sin(\eta_s y) + D\cos(\eta_s y)$$

- Horizontal and vertical displacement:

$$u = [ikf(y) - g'(y)]\exp[ik(x-cy)], \quad v = [f'(y) + ikg(y)]\exp[ik(x-cy)]$$

- Stress components:

$$\sigma_{xx} = -\mu\left[(k^2\gamma^2 + \xi^2\eta_d^2)f + 2ikg'\right]\exp[ik(x-cy)]$$
$$\sigma_{yy} = -\mu\left[(\xi^2k^2 + \gamma^2\eta_d^2)f - 2ikg'\right]\exp[ik(x-cy)]$$
$$\sigma_{xy} = \mu\left[\xi^2(\eta_s^2 - k^2)g + 2ikf'\right]\exp[ik(x-cy)]$$
$$\sigma_{zz} = \nu(\sigma_{xx} + \sigma_{yy}) = -\mu\xi^2 f(k^2 + \eta_d^2)\exp[ik(x-cy)]$$

- Constants:

$$\gamma^2 = c_d^2/c_s^2, \quad \xi^2 = \lambda/\mu$$

λ, μ are Lamé parameters, c_s, c_l are transverse and longitudinal wave celerities.

[1] **(Miklowitz, 1980)** The Theory of elastic waves and waveguides. North-Holland

Formal analysis II

- Prescribed displacements on boundary $y = H$:

$$\begin{cases} u(H) = ikf(H) - g'(H) = 0, \\ v(H) = f'(H) + ikg(H) = 0, \end{cases}$$

- Zero vertical displacement on boundary $y = 0$ (no opening):

$$v(H) = f'(0) + ikg(0) = 0$$

- Frictional relation between normal and shear tractions:

$$\sigma_{xy}(0) + F\sigma_{yy}(0) = \xi^2(\eta_s^2 - k^2)g(0) + 2ikf'(0) - F\left[(\xi^2k^2 + \gamma^2\eta_d^2)f(0) - 2ikg'(0)\right] = 0$$

- Obtain a linear system of equations for $X = \{A, B, C, D\}$

$$KX = 0$$

- For nontrivial solutions, we require that

$$\det(K) = 0$$

Formal analysis III

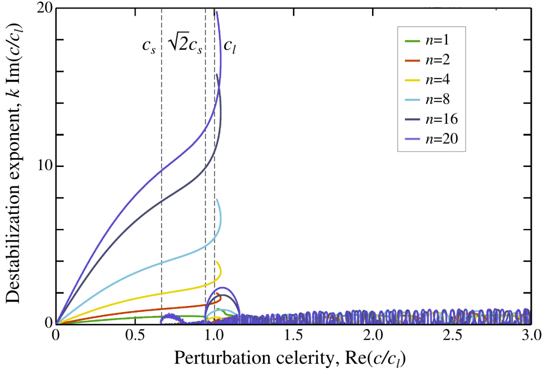
- $\det(K) = 0$ corresponds to this transcendental equation:

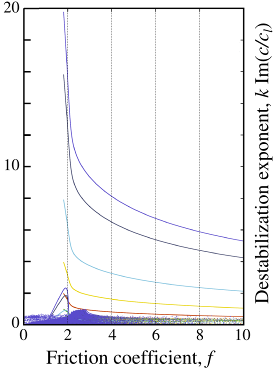
$$\boxed{\begin{aligned}&F\left[\sin(\eta_d H) + \frac{\eta_s\eta_d}{k^2}\sin(\eta_s H)\right]\left(-2\frac{\eta_d\eta_s}{k^2}\sin(\eta_d H) + (\xi^2 + \gamma^2\frac{{\eta_d}^2}{k^2})\sin(\eta_s H)\right) -\\ &- F\frac{\eta_s\eta_d}{k^2}\left[\cos(\eta_d H) - \cos(\eta_s H)\right]\left(2\cos(\eta_d H) + (\xi^2 + \gamma^2\frac{{\eta_d}^2}{k^2})\cos(\eta_s H)\right) +\\ &+ i\frac{\eta_d}{k}\left[2 + \xi^2\left(\frac{{\eta_s}^2}{k^2} - 1\right)\right]\left(\cos(\eta_d H)\sin(\eta_s H) + \frac{\eta_s\eta_d}{k^2}\cos(\eta_s H)\sin(\eta_d H)\right) = 0\end{aligned}}$$

- Fix ν and $k = 2\pi n/L$ with $n \in \mathbb{Z}$
- Take $\eta_d = |k|\sqrt{(c/c_d)^2 - 1}$, $\eta_s = |k|\sqrt{(c/c_s)^2 - 1}$
- Express $F = F(\mathrm{Re}(c), \mathrm{Im}(c))$
- Search for $\mathrm{Re}(F)$ at lines with $\mathrm{Im}(F) = 0$
- Solution $u \sim \exp[ik(x - \mathrm{Re}(c)t] \cdot \exp[k\mathrm{Im}(c)t]$

Formal analysis III

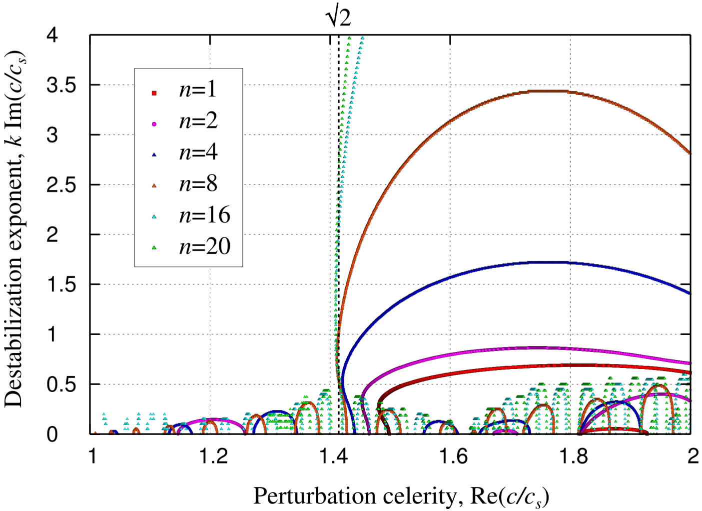
- Example: $\nu = 0.1$, $L = 2H$, $k = 2\pi n/L$





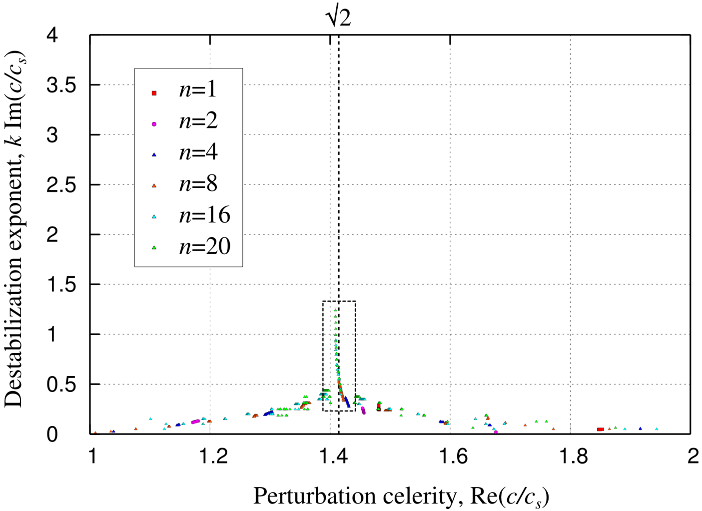
Formal analysis III

- Example: $\nu = 0.2$, $L = 2H$, $k = 2\pi n/L$



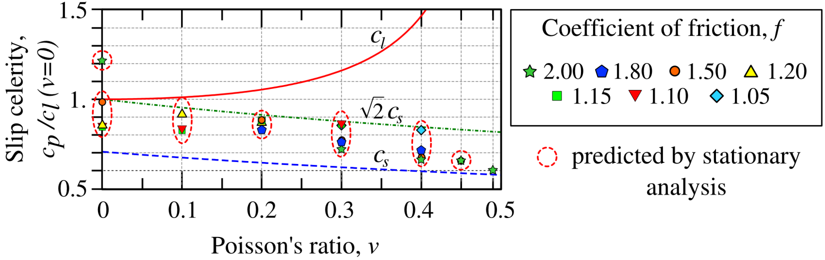
Formal analysis III

- Example: $\nu = 0.2$, $L = 2H$, $k = 2\pi n/L$

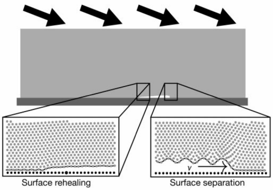


Formal analysis III

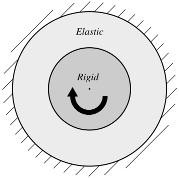
- Example: $\nu = 0.2$, $L = 2H$, $k = 2\pi n/L$



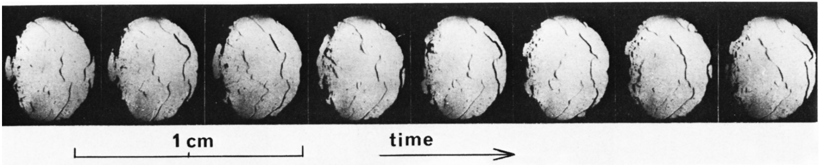
Results IV: references



(Gerde & Marder, 2001) Friction and Fracture, *Nature*



(Moirot, Nguyen, Oueslati, 2002) An example of stick-slip and stick-slip-separation waves, *Eur J Mech A-Solid*



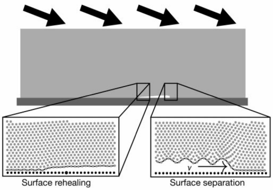
(Schallamach, 1971) How does rubber slide?, *Wear*

Remark: Schallamach waves are much slower.

(Comninou & Dundurs, 1977, 1978) Elastic interface waves involving separation

(Freund, 1978) Discussion: elastic interface waves involving separation

Results IV: references



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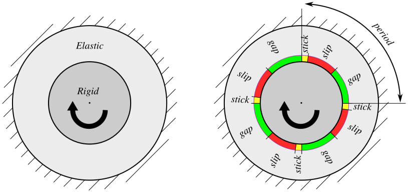

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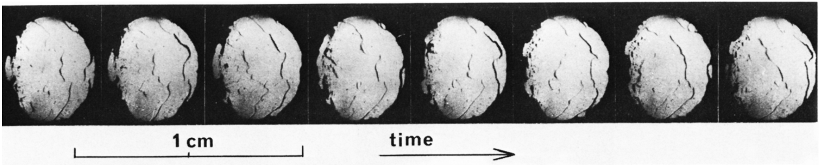

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(Freund, 1978) Discussion: elastic interface waves involving separation

Conclusion

- **Sub-critical regime**
  - stationary solution
  - supersonic stick-slip
  - or standing waves stick-slip
  - velocity dependent global friction
- **Critical regime**
  - intersonic transient slip pulse
  - transforms into opening pulse
  - ⋆ sliding without slipping(!)
  - ⋆ inversion of frictional force(!)
  - stationary waveguide analysis predicts instability and slip velocity

[1] V.A. Yastrebov, Sliding Without Slipping Under Coulomb Friction: Opening Waves and Inversion of Frictional Force, *Tribology Letters* 62:1-8 (2016)

[2] V.A. Yastrebov, Elastodynamic frictional sliding of an elastic layer on a rigid flat, *in preparation*

☺ Thank you for your attention!